Data Mining Distributed Streams

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Amazon Web Services
Single machine data processing
Distributed storage
Distributed compute (map/reduce, MPI, ...)

The World

Data + Compute

Data + Compute

Data + Compute

Data + Compute

Data + Compute

Data + Compute

Data + Compute

Computation

Result

Distributed compute (map/reduce, MPI, ...)
Distributed model (indexes, databases, Spark...)

The World

Data + Compute

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Data + Compute

Data + Compute

Query → Computation → Computation → Result
207 big-data infographics (a meta infographic)
Amazon Kinesis Analytics
The streaming model

The World

Compute

Sketch

Query Algorithm

Query

Result

Result
The distributed streaming model
The streaming model (more accurately)

$O(n)$ Items

Iterator

$O(\text{polylog}(n))$ Space

Sketch

$O(\text{polylog}(n))$ Computation per item

Computation

Result
Communication complexity

Iterator

1 7 8 1

Sketch

Iterator

0 1 7 7

Result
What Can we do in this model?

**Items**
(words, IP-addresses, events, clicks,...)
- Item frequencies
- Approximate Quantiles
- Counting distinct elements
- Moment and entropy estimation
- Approximate set operations
- Sampling

**Vectors**
(text documents, images, example features,...)
- Dimensionality reduction
- Clustering (k-means, k-median,...)
- Linear Regression
- Machine learning (some of it at least)

**Matrices**
(text corpora, recommendations, ...)
- Covariance estimation matrix
- Low rank approximation
- Sparsification

**Graphs**
(social networks, communications, ...)
- Connectivity
- Cut Sparsification
- Weighted Matching
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Frequency Counting


Demaine, Lopez-Ortiz, Munro. Frequency estimation of internet packet streams with limited space, 2002


The name "Lossy Counting" was used for a different algorithm by Manku and Motwani, 2002

Metwally, Agrawal, Abbadi, Efficient Computation of Frequent and Top-k Elements in Data Streams, 2006

Charikar, Chen, Farach-Colton, Finding frequent items in data streams, 2002

Cormode, Muthukrishnan, An Improved Data Stream Summary: The Count-Min Sketch and its Applications.
Problem Definition

\[ f(\text{box}) = 5 \]

\[ |f' - f| < \varepsilon n \]
Can we do better than sampling?

\[ f'(\) = 3 \cdot \frac{n}{\ell} \]

\[ \ell = \tilde{O}\left(\frac{1}{\varepsilon^2}\right) \]
\[ f'(\text{green}) = 2 \]

\[ f'(\text{black}) = 0 \]
Analysis

First fact: \( f'(x) \leq f(x) \)

Assume we delete \( t \) times

Second fact: \( f'(x) \geq f(x) - t \)

Therefore: \( |f'(x) - f(x)| \leq t \)
Analysis

Third fact: \( t \leq n/\ell \)

We delete \( \ell \) different items every time!

We get that: \( |f'(x) - f(x)| < \varepsilon n \)

When: \( \ell = 1/\varepsilon \) (much better than sampling!)
Analysis

Items’ exact probability \( p(x) = \frac{f(x)}{n} \)

Approximate probability \( p'(x) = \frac{f'(x)}{n} \)

We get:
\[
|p'(x) - p(x)| \leq \frac{1}{\ell}
\]

If \( \ell = 10,000 \) we get only a 0.01% error in our estimations.

We would need 10 \text{ billion} samples to get the same accuracy!
Email threads

A simple email thread (that’s not very hard to do...)
Threading Machine Generated Email

Ailon, Karnin, Maarek, Liberty, Threading Machine Generated Email, WSDM 2013
Threading Machine Generated Email

PayPal.com: "You submitted an order in the amount of * usd to overstock.com."

overstock.com: "Overstock.com password reset request."

payless.com: "Order confirmation"

C=632
w=1,221

overstock.com: "Your overstock.com order has shipped."

C=1,742
w=6,446

payless.com: "Your order is shipped"

C=193
w=12,098

C=652
w=1,300

C=753
w=1,395

C=769
w=1,490

C=153
w=704
Threading Machine Generated Email

- Order Confirmation (retail) 64% → Shipping Notification
- 19% back to Order Confirmation (retail)
- Utility bill payment due 44% → Payment received
- 35% back to Utility bill payment due
- 53% to Service cancelation
- 15% to Service cancelation
Streaming quantiles

Manku, Rajagopalan, Lindsay. Random sampling techniques for space efficient online computation of order statistics of large datasets.
Munro, Paterson. Selection and sorting with limited storage.
Greenwald, Khanna. Space-efficient online computation of quantile summaries.
Wang, Luo, Yi, Cormode. Quantiles over data streams: An experimental study.
Greenwald, Khanna. Quantiles and equidepth histograms over streams.
Agarwal, Cormode, Huang, Phillips, Wei, Yi. Mergeable summaries.
Felber, Ostrovsky. A randomized online quantile summary in $O(\frac{1}{\varepsilon} \log(1/\varepsilon))$ words.
Lang, Karnin, Liberty, Optimal Quantile Approximation in Streams.
Problem Definition

\[ R(\text{block}) = 0.6 \cdot n \]

Sampling \( \tilde{O}(1/\varepsilon^2) \) values gives \( |R' - R| < \varepsilon n \) can we do better?
The basic buffer idea

Buffer of size $k$

| 5 | 1 | 4 | 7 | 0 | 3 |
The basic buffer idea

Stores $k$ stream entries

3
0
7
4
1
5
The basic buffer idea

The buffer sorts $k$ stream entries

- 7
- 5
- 4
- 3
- 1
- 0
The basic buffer idea

Deletes every other item
The basic buffer idea

And outputs the rest with double the weight
The basic buffer idea

\[ R(x) = 2 \]

\[ R'_1(x) = 2 \]

\[ R'_2(x) = 2 \]

\[ R'_3(x) = 2 \]

\[ R_0(x) = 2 \]

\[ R'_1(x) = 6 \]

\[ R'_2(x) = 4 \]

\[ R'_3(x) = 4 \]

\[ x \]

\[ x \]
The basic buffer idea

Repeat $n/k$ time until the end of the stream

$|R'(x) - R(x)| < n/k$
Manku-Rajagopalan-Lindsay (MRL) sketch

\[
\log_2(n) \text{ Buffers of size } k
\]

\[
| R'(x) - R(x) | \leq n \log_2(n)/k
\]
Manku-Rajagopalan-Lindsay (MRL) sketch

If we set \( k = \log_2(n)/\varepsilon \)

We get \( |R'(x) - R(x)| \leq \varepsilon n \)

And we maintain only \( \log_2^2(n)/\varepsilon \) items from the stream!
Greenwald-Khanna (GK) sketch

Uses a completely different construction

It gets \[ |R'(x) - R(x)| \leq \varepsilon n \]

And maintains only \[ O(\log(n)/\varepsilon) \] items from the stream!
Agarwal, Cormode, Huang, Phillips, Wei, Yi (1)

\[ \log\left(\frac{1}{\varepsilon}\right) \text{ Buffers of size } k \]

start sampling after \( O\left(\frac{1}{\varepsilon^2}\right) \) items

Reduces space usage to \( \log^2(1/\varepsilon)/\varepsilon \) items from the stream.
Agarwal, Cormode, Huang, Phillips, Wei, Yi (2)

\[ R(x) = 1 \]

5 \hspace{1cm} 7

\[ R'(x) = 2 \]

5 \hspace{1cm} 7

\[ R'(x) = 0 \]

5 \hspace{1cm} 7

\( x \)

Reduces space usage to \( \log^{3/2}(1/\varepsilon)/\varepsilon \) items from the stream.

\[ R'(x) \] is a random variable now and

\[ E[R'(x)] = R(x) \]
Reduces space usage to $\sqrt{\log(1/\varepsilon)/\varepsilon}$ items from the stream.
Lang, Karnin, Liberty (2)

Exponentially decreasing buffer sizes

GK Sketch

Reduces space usage to $\log \log(1/\varepsilon)/\varepsilon$ items from the stream.

Which is Optimal!
Some experimental results
Count Distinct
(Demo Only)

sketches-core
Core Sketch Library.

GitHub, Inc. [US]  https://github.com/dasketches

Java  415  119  Updated a day ago
Assume you need to estimate the number of *unique* numbers in a file

```
>> head data.csv
0
1
0
0
3
0
2
3
7
3
3
2
```

In this one, row i tasks a value from [0,i] uniformly at random.
Some stats: there are 10,000,000 such numbers in this ~76Mb file.

```
>>time wc -lc data.csv
10000000 76046666 data.csv
real 0m0.101s
user 0m0.072s
sys 0m0.021s
```

Reading the file take ~1/10 seconds. We don’t foresee IO being an issue.
To count the number of distinct items you might try this:

```
$ sort data.csv | uniq | wc -l
```

However, it is faster to have “uniqify” while sorting.

```
$ sort data.csv -u | wc -l
```

```
$ time sort data.csv -u | wc -l
5001233
real 2m37.071s
user 2m36.587s
sys 0m0.376s
```
Still, most of the time is spent on comparing strings.

```
>> sort data.csv -u -n -S 100% | wc -l
```

This is much better!

```
>> time sort data.csv -u -n | wc -l
5001233
real 0m11.809s
user 0m11.587s
sys 0m0.228s
```
This is the way to do this with the sketching library

```
>> sketch uniq data.csv
```

```
>> time sketch uniq data.csv
Estimate : 4974249
Upper Bound : 5116569
Lower Bound : 4835874
```

real 0m1.527s
user 0m1.506s
sys 0m0.152s

Too fast to use the system monitor UI...

It uses ~ 32k of memory!
Thank you!